Impact of separately located obstacles on air flow movement calculation

A. Salokhiddinov, *A.* Savitskiy, *O.* Ashirova, *R.* Razzakov

"TIIAME" National Research University, Uzbekistan

Abstract. In hydrodynamic calculations, the confederation of the right set of coordinates of a continuous medium (gas, liquid) becomes complicated because it makes the calculation mechanism-specific and tied only to a specific object. The paper is devoted to an improved method to calculate the small-scale circulation of the surface layer of the atmosphere in urban areas with many isolated buildings from each other. The proposed method will work for explicit and implicit schemes for calculating the transport equations and conservation of momentum of motion. It is designed to calculate the small-scale circulation of the surface layer of the atmosphere in urban areas with many isolated buildings from each other. The method allows for consideration of the interaction of a continuous medium flow with impenetrable obstacles discretely and randomly located inside the calculation area. This method will be helpful when calculating microcirculation in urban development.

1 Introduction

In large-scale atmospheric circulations, we can assume that there are no physical obstacles to the movement of atmospheric flows. Medium-scale models describing the movement of atmospheric flows in the surface layer should consider obstacle ranges. Usually, the calculation area is positioned so that the obstacle is located on its border. Suppose it is required to conduct hydrodynamic calculations in reservoirs with a rugged coastline or small-scale atmospheric circulation in urban development. In that case, the problem of correct algorithmization of boundary conditions arises. The right set of coordinates in which the motion of a continuous medium (gas, liquid) should or should not be considered becomes a problem. This problem is complex for explicit schemes for calculating momentum, matter conservation, and transfer equations. But this problem becomes complicated if implicit calculation schemes are used. Suppose the line we calculate according to implicit schemes (usually a forward and reverse run) intersects several mysterious islands or buildings. In that case, the line splits into separate fragments, and we should carry each separate and unrelated calculation out with the rest of the segments. Of course, we can carefully enter all the coordinates of buildings or islands into the calculation mechanism of a particular object. But this approach makes the calculation mechanism-specific and tied only to a specific object.

2 Materials and methods

We propose a method to account for the impact of multiple obstacles on moving air flows using some imaginary stationary phase with special properties. We tested the method using a hydrodynamic calculation in the calculation area in which there are obstacles impervious to the flow. With this method, the most complex urban development is easily algorithmized when calculating air flows moving through this building. The proposed method will work both for explicit schemes and for implicit schemes for calculating the equations of transport and conservation of momentum of motion. This approach is specially designed to calculate the small-scale circulation of the surface layer of the atmosphere in urban areas with many isolated buildings from each other.

3 Results and Discussion

Boundary conditions of two types usually reach the boundaries of the solution area of aerohydrodynamic problems (Tsvetova, 1999; Wendroff, 1968). On solid non-permeable surfaces bounding the calculation area, the so-called "sticking" boundary condition for the velocity vector is set.

$$\vec{V}\Big|_{i,j,k\in G} = 0 \qquad (1)$$

where,
$\vec{V}$ is velocity vector,
i, j, k are integer coordinates of the grid function,
G is the surface bounding the calculation area.

On the part of the surface through which gas or liquid enters/leaves, the value of the velocity vector is set (or calculated by any procedures external to the primary model), which may depend on time and may depend on the solution itself. Usually, this vector is always deterministic and is calculated using parameters or calculation results known at the time of calculation. We can always consider this vector as a given (deterministic) one.

$$\vec{V}\Big|_{i,j,k\in G} = \vec{U}(t) \qquad (2)$$

where,
$\vec{U}(t)$ is the velocity vector known for each calculated time step.

On the section of the surface covering the calculation area filled with gas or liquid from the area also filled with gas or liquid and not attractive to the calculator and excluded from the calculation by his subjective decision, the tangential and normal stresses in the gas medium are set to zero. That is, the transmission of the momentum of motion through the surface is equated to zero or determined by empirical formulas when there is some transmission of the momentum of movement in one direction or the other. For example, the impact of the wind blowing outside the calculation area must be addressed.

$$\vec{\tau}\big|_{i,j,k\in G^-} = \vec{f}(t)\Big|_{i,j,k\in G^+} \qquad (3)$$

where,
$\vec{\tau}$ is vector of stresses acting in a gas/liquid on a surface G,
$\vec{f}(t)$ is a well-known vector function in most problems of hydromechanics calculated based on empirical formulas linking the tangential stress on the water surface with the wind speed at some fixed height.

The boundary conditions for the impurity transported by a gas or liquid stream are more complicated. Still, they are derived as formulas based on the initially constructed boundary conditions (Kochin, N.E., Kibel, I.A., and Rose, N.V., 1963; Kordzadze, 2017; Kulesh V.P. et al., 1971) for the velocity fields of the gas-liquid flow and the conservation laws of the transported substance.

When the calculation takes place inside the volumes of space with simple shapes, there are no particular problems. It is straightforward to calculate objects where any coordinate has one entry point into the calculation area and an exit point from the calculation area. For example, the Black Sea case (Lonin S.A., 1990; Marchuk G.I. et al., 1975). But everything changes if the Baltic Sea is calculated with a very complex configuration of shores (Kulesh V.P. et al., 1971; Sustavov Yu.V. & Chernysheva E.C., 1980) or even such an object as Lake Baikal (Marchuk & Sarkisyan, 1988; Tsvetova, 1999; Tsydenov, B. O. and Starchenko; A. V., 2011), but with the need to consider the interaction with the water masses of Olkhon Island.

Microcirculation of the surface layer of the atmosphere in urban conditions is much more complex compared to both the Baltic Sea and Lake Baikal case to set boundary conditions. Free-standing buildings of different heights turn the calculation area into a complicated maze. It is a considerable problem to track and correctly set the coordinates of the location of points that limit the calculation area from zones that are not subject to calculation but are entirely covered by the calculation area.

This problem is also difficult for explicit calculation schemes but becomes much more complicated when using implicit calculation schemes. In this case, the line along which the calculation is carried out splits into segments. In the case of implicit calculation schemes and the run-through method, it is necessary to calculate components separately on all coordinate lines in all coordinate directions (Marchuk, 1974). At the same time, it is required to correctly track and calculate or keep in the machine's memory the coordinates of the beginning and end of each calculated segment on the lines crossing the buildings entirely located inside the calculation area. Algorithmization of finding the beginning and end of each segment is a separate complex task. Usually, it is not even solved by turning each mathematical model into a unique calculation mechanism tied to some specific object.

Description of the method of accounting for the interaction of arbitrarily placed obstacles and airflow in the calculation zone. Suppose the entire calculation area is filled with some imaginary phase with special properties:

1. The entire calculation area contains some imaginary phase;
2. The imaginary phase does not move, and the speed of movement is zero;
3. The amount of space occupied by the imaginary phase is zero;
4. If the imaginary phase is located inside an impenetrable object, then the specific area of contact between the mythical phase and the airflow is given by a significant value;
5. If the imaginary phase is located in a zone where the airflow moves freely without encountering buildings or other structures, then the specific contact area between the mythical phase and the airflow is set to zero.

According to property 1, it is easy to determine the continuous phase distribution field by setting property 4 or property 5 in each coordinate of the calculation area, according to the layout of buildings, fences, and other impenetrable obstacles for airflow movement.

According to property 3, the existing terms of the equation of motion of a gas (liquid) do not change. Indeed, when considering multiphase flows, multipliers equal to the relative volume of each phase appear before the terms in the equations of motion. But if the volume of the imaginary and added phase is zero, then in the main and calculated phase equations, the multiplier will equal one. The equation for the mythical phase will contain terms equal to zero because of their multiplication by zero (Rakhmatulin, K.A., 1956).

Properties 4 or 5 force us to add to the equation the motion of a gas or liquid; the term is responsible for the force of interfacial friction. Furthermore, this friction can be equal to zero according to property 5 or be a vast amount capable of stopping the movement of a gas or liquid according to property 4.

We investigate the effectiveness of this approach of describing calculation areas with complex configurations of boundaries and obstacles by the example of calculating the movement of airflow through the calculation area with randomly located obstacles that simulate urban development in the test problem.

A moving medium under the influence of a pressure drop is assumed to be incompressible. The compressibility or incompressibility of the medium in this study is not a fundamental condition. The equations describing the motion of a continuous incompressible viscous medium in three-dimensional space (Kochin, N.E., Kibel, I.A., and Rose, N.V., 1963; Landau L.D. and Lifshitz E.M., 1986) and after the minor simplifications are presented in the system of equations (4).

$$\frac{d\vec{V}}{d\,t} = \vec{f} - \frac{1}{\varrho}\cdot \mathrm{grad}\,\mathrm{P} + A\cdot\vec{V} + \nu\cdot\Delta\vec{V} \quad (4)$$
$$div\,\vec{\mathbf{V}} = 0$$

Where $\vec{V}$ is vector of motion of an air-vapor (continuous) medium with components V_x, V_y, V_z,

d is full derivative,

x, y, z are spatial coordinates,

P is pressure,

ϱ is the density of the air vapor medium,

$\vec{f}$ is mass force with components f_x, f_y, f_z,

A coefficient equal to zero in the calculation area where there are no obstacles to the movement of the airflow and equal to a significant value (in the test problem, it is assumed to be equal to 100) when the calculated coordinate point of the solution area is located on the surface or inside the obstacle to the movement of the airflow,

ν is kinematic viscosity.

$\mathrm{grad}(*), div(*), div\,\mathrm{grad}(*) = \Delta\,(*)$ are differential operators: gradient, divergence, and Laplacian, respectively.

For simplicity, we will solve the two-dimensional problem of flowing around a group of obstacles with a continuous flow using the imaginary phase accounting mechanism with the properties described above.

To organize the calculation algorithm, we apply the approach common in aero hydrodynamics (Kordzadze, 2017; Marchuk & Sarkisyan, 1988; Mingalev V.S. et al., 2015; Savitsky A.G., 1981; Wang & Kravitz, 1980). This approach considers vertical velocities and accelerations minor compared to horizontal accelerations and velocities. This makes it possible to simplify the vertical component of the equations of motion of a continuous medium (gas, liquid) to the level of hydrostatics. From the joint solution of the non-compressibility equation of the medium written in the second line of the system of equations (4) and the hydrostatic equation, to which the vertical component of the momentum conservation equation can be simplified, a function follows to determine some isosurface that creates hydrostatic pressure in a moving medium.

$$\frac{\frac{\partial\psi}{\partial t}}{|grad\,\psi|} + \vec{V}\cdot\vec{n} = \mathbf{0} \quad (5)$$
$$\psi = z - z_p = const$$

where:
ψ is a series of isosurfaces defined by the equality $\psi = z - z_p$, for $const = 0$, $Z\ coordinate$ in the series of isosurfaces coincides with the location of the free surface,
$\vec{n}$ is a single normal vector on an isosurface directed vertically upwards,
z is vertical coordinate,
z_p is the vertical coordinate on the isosurface, which depends on the coordinates "x" and "y" and the time t.
The system of equations of motion of a continuous medium acquires a form convenient for algorithmization (6), which is greatly simplified under the assumption that the isosurface $\psi = z - z_p = 0$ coincides with the movable surface of the calculation area providing hydrostatic pressure, and this isosurface is close to horizontal.

$$\frac{\partial \psi}{\partial t} = V_z \big|_{z=z_p} = \int_{z_d}^{z_p} \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \right) \cdot dx$$

$$\frac{d\,V_x}{d\,t} = -\frac{1}{\varrho_o} \frac{\partial Z_p}{\partial x} + A \cdot V_x + \nu \cdot \left(\frac{\partial^2 V_x}{(\partial x)^2} + \frac{\partial^2 V_x}{(\partial y)^2} \right) - K_f \cdot V_x$$

$$\frac{d\,V_y}{d\,t} = -\frac{1}{\varrho_o} \frac{\partial Z_p}{\partial y} + A \cdot V_y + \nu \cdot \left(\frac{\partial^2 V_y}{(\partial x)^2} + \frac{\partial^2 V_y}{(\partial y)^2} \right) - K_f \cdot V_y \quad (6)$$

where:
K_f is the coefficient of friction allows for considering the friction force between the moving flow and the underlying surface. The coefficient depends on the thickness of the flat flow moving over the underlying and stationary surface.

Note that the term containing appears due to averaging ("convolution," integration) of the horizontal components of the three-dimensional equations of motion (4) vertically and the application of the theorem on the average value of certain integrals. Explicit calculation schemes were used for the solution. The well-known approach (Roache, 1980) of writing inertial terms in divergent forms was used to ensure conservatism in transferring the momentum of motion. Note that this approach only applies to an incompressible continuous medium (Roache, 1980).
Figure 1 shows the velocity field of a continuous flow moving from left to right under the influence of a pressure drop between the left and right boundaries of the calculation area.

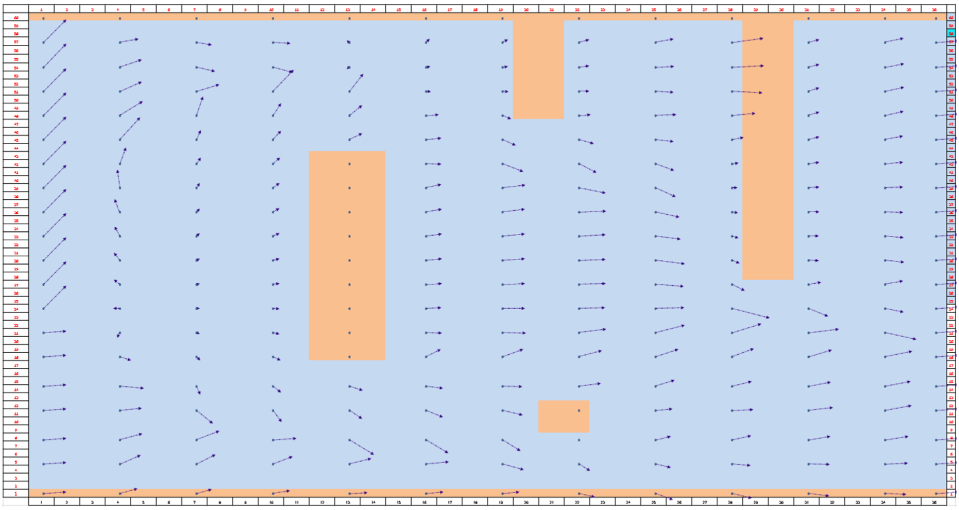
Figure 1. Velocity vector field in continuous stream flowing around discrete impenetrable obstacles.

Figure 1 shows how the flow bypasses impenetrable obstacles for it. In this case, the flow forms zones of increased hydrostatic pressure in front of obstacles. We showed zones of increased hydrostatic pressure in Figure 2.

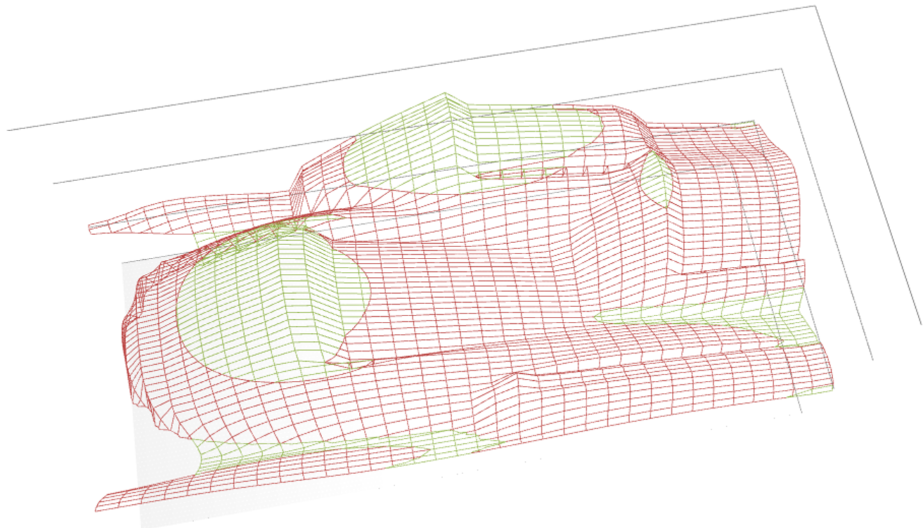

Figure 2. Zones of increased hydrostatic pressure in continuous stream flowing around discrete impenetrable obstacles.

Let's complicate the test problem by adding to the system of equations (6) the equation of conservation and transfer of a conservative impurity

$$S\frac{\partial S}{\partial t} + \frac{\partial (V_x \cdot S)}{\partial x} + \frac{\partial (V_y \cdot S)}{\partial y} = 0 \qquad (7)$$

We used a new conservative finite difference scheme developed at the TIIAME National Research University (Salokhiddinov, A. T. et al., 2022; Salokhiddinov et al., 2023) for calculation. I have repeatedly tested this scheme on many test problems, and it surpasses the most well-known finite difference schemes used to calculate similar aero hydrodynamics issues (Courant et al., 1967; Lax & Wendroff, 1960; Roache, 1980).

Let us set the concentration of the substance on the upper half of the left boundary of the solution area equal to one. Figure 3 shows the spread of the impurity over the calculation area, which contains obstacles simulating urban development. The influence of obstacles on the propagation field of a conservative impurity is visible.

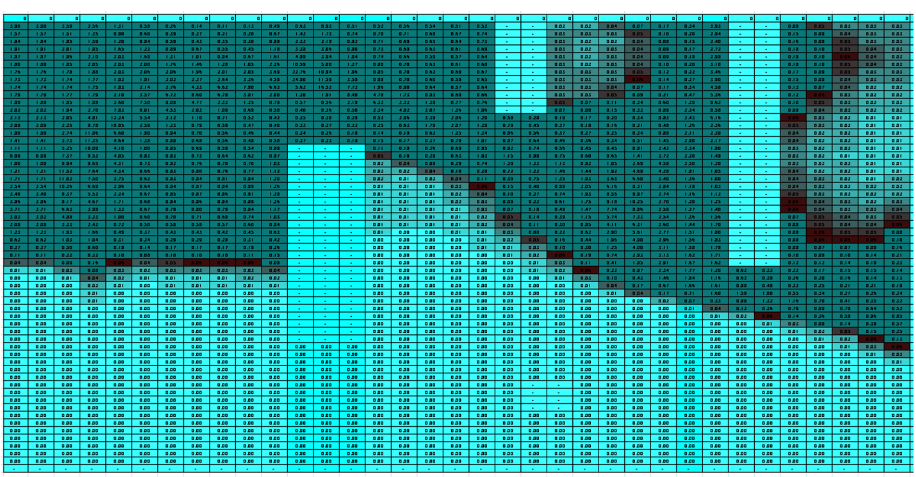

Figure 3. Distribution of conservative admixture in test problem of urban conditions.

4 Conclusions

A method is proposed for considering the interaction of a continuous medium flow with impenetrable obstacles discretely and randomly located inside the calculation area. This method will be especially useful when calculating microcirculation in urban development.

A method is considered and tested on a test example that makes it easy to describe the impact on the flow of a continuous medium: mysterious, multiple, isolated, and mysterious objects for the flow. Objects can be located entirely or partially inside the calculation area. The effect on the microcirculation of the surface layer of the atmosphere from dense urban development by detached buildings located inside the calculation area can be easily algorithmized by entering the calculation of some additional imaginary phase with some special properties. Reconfiguring the mechanism for calculating the movement of a continuous flow from object to object is simple and easy. Therefore, when using this approach to algorithmize the position of arbitrary obstacles, the mechanism for calculating the movement of air and water flows becomes universal.

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